

PREAMBULE

Compléter le tableau suivant :

x	$-\frac{5\pi}{6}$	$-\frac{3\pi}{4}$	$-\frac{2\pi}{3}$	$-\frac{\pi}{2}$	$-\frac{\pi}{3}$	$-\frac{\pi}{4}$	$-\frac{\pi}{6}$	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π
$\cos x$								1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0				
$\sin x$								0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1				

EXERCICE 1 - EQUATIONS DU TYPE $\cos x = \alpha$ **Remarque :** Si $\alpha > 1$ ou $\alpha < -1$, l'équation n'a aucune solution.**1. Si $\alpha = -1$** **Exemple :**

$$\cos 3x = -1$$

$$\Leftrightarrow \cos 3x = \cos \pi$$

$$\Leftrightarrow 3x = \pi + k2\pi, k \in \mathbb{Z}$$

$$\Leftrightarrow x = \frac{\pi}{3} + k\frac{2\pi}{3}, k \in \mathbb{Z}$$

$$\text{donc } x = \frac{\pi}{3}, \pi, \frac{5\pi}{3}, \frac{7\pi}{3}, 3\pi, \frac{11\pi}{3} \dots$$

2. Si $-1 < \alpha < 1$ **Exemple :**

$$\cos x = \frac{1}{2}$$

$$\Leftrightarrow \cos x = \cos \frac{\pi}{3}$$

$$\Leftrightarrow \begin{cases} x = \frac{\pi}{3} + k2\pi & k \in \mathbb{Z} \\ \text{ou} \\ x = -\frac{\pi}{3} + k2\pi & k \in \mathbb{Z} \end{cases}$$

$$\Leftrightarrow \begin{cases} x = \frac{\pi}{3} + k2\pi & k \in \mathbb{Z} \\ \text{ou} \\ x = -\frac{\pi}{3} + k2\pi & k \in \mathbb{Z} \end{cases}$$

$$\text{donc } x = \frac{\pi}{3}, \frac{7\pi}{3}, \frac{13\pi}{3}, \frac{-\pi}{3}, \frac{-7\pi}{3} \dots$$

3. Si $\alpha = 1$ **Exemple :**

$$\cos 2x = 1$$

$$\Leftrightarrow \cos 2x = \cos 0$$

$$\Leftrightarrow 2x = 0 + k2\pi, k \in \mathbb{Z}$$

$$\Leftrightarrow x = 0 + k\frac{2\pi}{2}, k \in \mathbb{Z}$$

$$\Leftrightarrow x = k\pi, k \in \mathbb{Z}$$

$$\text{donc } x = \pi, 2\pi, 3\pi, -\pi, -2\pi, -3\pi \dots$$

→ De la même façon, résoudre :

1. $\cos 2x = -1$

2. $\cos x = \frac{\sqrt{3}}{2}$

3. $\cos 5x = 1$

EXERCICE 2 - EQUATIONS DU TYPE $\sin x = \alpha$ **Remarque :** Si $\alpha > 1$ ou $\alpha < -1$, l'équation n'a aucune solution.**1. Si $\alpha = -1$** **Exemple :**

$$\sin 3x = -1$$

$$\Leftrightarrow \sin 3x = \sin \frac{-\pi}{2}$$

$$\Leftrightarrow 3x = \frac{-\pi}{2} + k2\pi, k \in \mathbb{Z}$$

$$\Leftrightarrow x = \frac{-\pi}{6} + k\frac{2\pi}{3}, k \in \mathbb{Z}$$

$$\text{donc } x = \frac{-\pi}{6}, \frac{5\pi}{6}, \frac{3\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6} \dots$$

2. Si $-1 < \alpha < 1$ **Exemple :**

$$\sin x = \frac{1}{2}$$

$$\Leftrightarrow \sin x = \sin \frac{\pi}{6}$$

$$\Leftrightarrow \begin{cases} x = \frac{\pi}{6} + k2\pi & k \in \mathbb{Z} \\ \text{ou} \\ x = \pi - \frac{\pi}{6} + k2\pi & k \in \mathbb{Z} \end{cases}$$

$$\Leftrightarrow \begin{cases} x = \frac{\pi}{6} + k2\pi & k \in \mathbb{Z} \\ \text{ou} \\ x = \frac{5\pi}{6} + k2\pi & k \in \mathbb{Z} \end{cases}$$

$$\Leftrightarrow \begin{cases} x = \frac{\pi}{6} + k2\pi & k \in \mathbb{Z} \\ \text{ou} \\ x = \frac{5\pi}{6} + k2\pi & k \in \mathbb{Z} \end{cases}$$

$$\Leftrightarrow \begin{cases} x = \frac{\pi}{6} + k2\pi & k \in \mathbb{Z} \\ \text{ou} \\ x = \frac{5\pi}{6} + k2\pi & k \in \mathbb{Z} \end{cases}$$

$$\text{donc } x = \frac{\pi}{6}, \frac{13\pi}{6}, \frac{25\pi}{6}, \frac{37\pi}{6} \dots$$

$$\text{ou } x = \frac{5\pi}{6}, \frac{17\pi}{6}, \frac{29\pi}{6}, \frac{41\pi}{6} \dots$$

3. Si $\alpha = 1$ **Exemple :**

$$\sin 2x = 1$$

$$\Leftrightarrow \sin 2x = \sin \frac{\pi}{2}$$

$$\Leftrightarrow 2x = \frac{\pi}{2} + k2\pi, k \in \mathbb{Z}$$

$$\Leftrightarrow x = \frac{\pi}{4} + k\frac{2\pi}{2}, k \in \mathbb{Z}$$

$$\Leftrightarrow x = \frac{\pi}{4} + k\pi, k \in \mathbb{Z}$$

$$\text{donc } x = \frac{\pi}{4}, \frac{9\pi}{4}, \frac{17\pi}{4}, \frac{25\pi}{4} \dots$$

→ De la même façon, résoudre :

1. $\sin 2x = -1$

2. $\sin x = \frac{\sqrt{3}}{2}$

3. $\sin 4x = 1$